

# On the generalized Hartman effect and transmission time for a particle tunneling through two identical rectangular potential barriers

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## Abstract

We develop a new quantum-mechanical model of scattering a particle on a one-dimensional (1D) system of two identical rectangular potential barriers, which treats this process as that consisting of two alternative subprocesses (transmission and reflection) and represents the wave function to describe the whole process as the superposition of a unique pair of components to describe causally evolving transmission and reflection at all stages of scattering. We define for each subprocess the asymptotic and local group scattering times as well as the dwell time. In the limit of an opaque system the *asymptotic* group transmission time saturates, while the dwell and *local* group transmission times increase exponentially and depend on the distance between the barriers.

**Keywords:** generalized Hartman effect, dwell time, local and asymptotic group time, Bohmian Wigner Feynman trajectory

## 1. Introduction

As is shown in [1, 2] for a particle to pass through a 1D system of two identical rectangular potential barriers, the tunneling time is independent in the opaque limit not only on the width of the barriers but also on the distance between them. This finding, known as the generalized Hartman effect, is evident to enforce the tension to appear due to the usual Hartman effect [3] between the conventional quantum model (CQM) of this process and special relativity. It says once more that either the CQM, which predicts this paradoxical effect for a (not virtual) particle, or the basic principle of special relativity, which forbids superluminal velocities, must be discarded.

Despite the statements (see, e.g., [4]) about experimental observations of the Hartman effect, we intend here to support the first and to present a new model of this process. The point is that the CQM conflict not only with special relativity but also with classical probability theory. At the same time the adequate quantum description of this process, as a basically statistical one, must be compatible with the latter at the macro-scales.

By probability theory, experimental data obtained for transmitted and reflected particles, with the help of two detectors placed on each side of the system, are mutually incompatible. They do not belong to a single Kolmogorovian probability space (see [5, 6]), what makes inadmissible any averaging over such data. Thus, quantum mechanics must treat these data in the same way. It must imply that the squared modulus of the state to describe this process in any representation is not the probability density (see also [7]), and the process itself is a complex one to consist of two causally evolved alternative subprocesses, transmission and reflection, to need their individual description

at all stages of scattering. The CQM is evident to violate these requirements of probability theory and special relativity.

With relation to these requirements all scattering times introduced within this model for a particle scattering on a 1D potential barrier (see reviews [10, 11, 12, 13, 14] as well as the recent papers [8, 9]) fall in fact into two classes. The first one includes scattering times which like the Büttiker dwell time  $\tau_D$  are based on the properties of the wave function in the *barrier* region. The second one includes scattering times which like the Wigner phase time  $\tau_{ph}$  characterize *scattered* wave packets.

The main feature of  $\tau_D$  is that this time quantity does not distinguish between transmitted and reflected particles, and thus it relates neither to transmission nor to reflection [10]. Sometimes (ibid)  $\tau_D$  is treated as the mean value of the transmission time  $\tau_T$  and the reflection time  $\tau_R$ :  $\tau_D = \tau_T \mathbf{T} + \tau_R \mathbf{R}$  where  $\mathbf{T}$  is the (real) transmission coefficient,  $\mathbf{R}$  is the reflection one;  $\mathbf{T} + \mathbf{R} = 1$ . But this status unfits  $\tau_D$ , because this averaging rule contradicts probability theory to forbid *any* averaging over statistical data associated with alternative subprocesses.

To show explicitly that this rule leads to nonphysical results, it is sufficient to calculate on its basis the mean value of the particle's momentum for the final stage of a 1D completed scattering, provided that at its first stage a particle has the definite nonzero momentum  $p_0$  and, besides,  $\mathbf{T} = \mathbf{R} = 1/2$ . It is evident that the searched-for mean value is zero in this case, which has nothing to do both with the momentum  $p_0$  of transmitted particles and with the momentum  $-p_0$  of reflected ones.

Thus, by probability theory, no physical meaning can be attributed to the dwell time  $\tau_D$ . As regards  $\tau_{ph}$ , at first glance it is this quantity that gives the transmission time. However, this is not the case. To treat this quantity (see [1, 2, 3, 9]) as the time spent by transmitted particles in the *barrier* region (i.e., as a *local* transmission time) is erroneous, because  $\tau_{ph}$  is defined via the amplitude and phase of the transmitted wave packet to

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move *far* from this region. To treat  $\tau_{ph}$  (see [10]) as the *asymptotic* (extrapolated) transmission time is erroneous, because this concept violates the causality principle.

Indeed, to define the asymptotic (group) transmission time, one has to compare the motion of the center of "mass" (CM) of the transmitted wave packet with that of the reference wave packet (RWP). The latter, by definition, moves freely at all stages of scattering and coincides at the first stage with the wave packet to evolve causally into the transmitted one. The searched-for quantity is simply the local group transmission time defined for the RWP's CM plus the time delay of the CM of the transmitted wave packet relative to the CM of the RWP, acquired by the former in the asymptotically large spatial region, in the course of the whole 1D completed scattering.

However, in the phase time concept the role of the RWP is played by the wave packet to coincide at the first stage of scattering with the incident one to describe the whole ensemble of particles (but with the norm reduced *ad hoc* to  $\mathbf{T}$ ). Such a choice is evident to be erroneous since "...an incoming peak or centroid does not, in any obvious physically causative sense, turn into an outgoing peak or centroid..." [11].

The same concerns the conditional transmission dwell time defined for the stationary state as a weak value (see [15] as well as [9]). As is assumed in [15], the final transmitted wave and the corresponding initial one are related "...by a parity flip combined with a time reversal...". But the latter does not evolve causally into the former. There is no spatial point where these two complex-valued (incoming and outgoing) waves could be joined, resulting in the stationary wave function, everywhere continuous together with the corresponding probability current density (see also Section 3).

Thus, the CQM allows one to define the average time of *arrival* of transmitted particles at some remote point in the transmission region, but to define unambiguously their *departure* time is impossible in principle within this model. Figuratively speaking, the CQM gives no rule for setting the position of the clock's hand, which would correspond to the beginning of the transmission subprocess. As a result, none of the transmission time concepts introduced in the CQM yields asymptotic and, all the more, local ones; what makes impossible the unambiguous interpretation of their experimental realizations.

Solving this problem needs the individual description of transmission and reflection at all stages of scattering. This was realized in [16] by the example of symmetric potential barriers, where the time-dependent scattering state to describe a 1D completed scattering was uniquely divided into two causally evolved substates to describe the alternative subprocesses.

This approach reconciles the quantum-mechanical superposition principle with the "either-or" rule to govern mutually exclusive events in classical probability theory. It allows one to introduce correctly both the asymptotic and local scattering times for a 1D completed scattering, as well as to shed new light on the usual and generalized Hartman effects. To perform this program, we have to dwell also on the key points of our approach [16] in order to make this paper readable on its own.

## 2. Backgrounds

Let a particle with a definite energy  $E$  ( $E > 0$ ) impinge from the left on the system of two identical rectangular barriers of height  $V_0$  ( $V_0 \geq E$ ), which are located in the finite intervals  $[a_1, b_1]$  and  $[a_2, b_2]$ ;  $b_1 - a_1 = b_2 - a_2 = d$  is the barriers width;  $a_2 - b_1 = l$  is the distance between the barriers;  $b_2 - a_1 = D$ . This system is symmetric and hence the approach [16] is applicable here. The only difference is that now we deal with the potential to be not smooth inside the region  $[a_1, b_2]$ ; the intervals  $[a_1, b_1]$ ,  $[b_1, a_2]$  and  $[a_2, b_2]$  should be handled separately.

Let  $\Psi_{full}(x, k)$  denote the stationary state of the ensemble of particles to take part in the process;  $k = \sqrt{2mE}/\hbar$ . To the left and to the right of the system we have

$$\begin{aligned}\Psi_{full}(x, k) &= e^{ikx} + b_{out}e^{ik(2a_1-x)}, & x \leq a_1; \\ \Psi_{full}(x, k) &= a_{out}e^{ik(x-D)}, & x \geq b_2; \\ a_{out} &= \frac{1}{2} \left( \frac{Q}{Q^*} - \frac{P}{P^*} \right), & b_{out} = -\frac{1}{2} \left( \frac{Q}{Q^*} + \frac{P}{P^*} \right); \\ Q &= q^* e^{i\phi} + ip e^{-i\phi}, & P = iq^* e^{i\phi} + p e^{-i\phi}; \\ q &= \frac{e^{-iJ}}{\sqrt{T}}, & p = \sqrt{\frac{R}{T}}, & \phi = \frac{kl}{2};\end{aligned}\tag{1}$$

$T$  is the transmission coefficient ( $R = 1 - T$ ) and  $J$  is the phase, which characterize each rectangular barrier of this system:

$$\begin{aligned}T &= \left[ 1 + \theta_{(+)}^2 \sinh^2(kd) \right]^{-1}, & J &= \arctan(\theta_{(-)} \tanh(kd)) \\ \theta_{(\pm)} &= \frac{1}{2} \left( \frac{k}{\kappa} \pm \frac{\kappa}{k} \right), & \kappa &= \sqrt{2m(V_0 - E)}/\hbar.\end{aligned}$$

Inside the  $n$ -th barrier region  $[a_n, b_n]$  ( $n = 1, 2$ )

$$\begin{aligned}\Psi_{full} &= a_{full}^{(n)} \sinh[\kappa(x - b_n)] + b_{full}^{(n)} \cosh[\kappa(x - b_n)]; \\ a_{full}^{(1)} &= ia_{out} \frac{k}{\kappa} (qe^{-ikl} + ip e^{ikl}) e^{ika_1}, \\ b_{full}^{(1)} &= a_{out} (qe^{-ikl} - ip e^{ikl}) e^{ika_1}, \\ a_{full}^{(2)} &= ia_{out} \frac{k}{\kappa} e^{ika_1}, & b_{full}^{(2)} &= a_{out} e^{ika_1}.\end{aligned}\tag{2}$$

In the gap between the barriers ( $b_1 \leq x \leq a_2$ )

$$\begin{aligned}\Psi_{full} &= a_{full}^{gap} \sin[k(x - x_c)] + b_{full}^{gap} \cos[k(x - x_c)]; \\ a_{full}^{gap} &= -a_{out} P^* e^{ika_1}, & b_{full}^{gap} &= a_{out} Q^* e^{ika_1}.\end{aligned}\tag{3}$$

## 3. Wave functions for transmission and reflection

As was shown in [16], the state  $\Psi_{full}(x, k)$  can be *uniquely* divided into two components to describe causally evolving transmission and reflection. Namely, there is a unique pair of the wave functions  $\psi_{tr}(x, k)$  and  $\psi_{ref}(x, k)$  which obey equation

$$\psi_{tr}(x, k) + \psi_{ref}(x, k) = \Psi_{full}(x, k)\tag{4}$$

as well as possess the following properties: (a) either function unlike  $\Psi_{full}(x, k)$  has one outgoing and one incoming wave; (b) the outgoing wave of  $\psi_{tr}(x, k)$  coincides with the transmitted wave, and that of  $\psi_{ref}(x, k)$  coincides with the reflected one;

(c) the incoming wave of either wave function is causally connected at the joining point  $x_c$  to the corresponding outgoing one – the (complex-valued) functions  $\psi_{tr}(x, k)$  and  $\psi_{ref}(x, k)$  as well as the corresponding probability current densities are continuous at this point (but the first derivative of either function is discontinuous here); (d) for any *symmetric* potential barrier,  $x_c$  coincides with the midpoint of its barrier region, irrespective of the particle's energy; i.e., for the system under consideration  $x_c = (b_2 + a_1)/2$ .

Beyond the interval  $[a_1, x_c]$  we have for  $x < a_1$

$$\psi_{tr}(x, t) = A_{in}^{tr} e^{ikx}, \quad \psi_{ref}(x, t) = A_{in}^{ref} e^{ikx} + b_{out} e^{ik(2a_1-x)};$$

for  $x \geq x_c$

$$\psi_{tr}(x, k) \equiv \Psi_{full}(x, k), \quad \psi_{ref}(x, t) \equiv 0.$$

That is, by this approach there is no interference in the region  $x \geq x_c$  – reflected particles merely do not cross the midpoint  $x_c$ . The amplitudes  $A_{in}^{tr}(k)$  and  $A_{in}^{ref}(k)$  read as

$$\begin{aligned} A_{in}^{tr} &= a_{out}^*(b_{out} + a_{out}) \equiv \sqrt{\mathcal{T}} (\sqrt{\mathcal{T}} \pm i \sqrt{\mathcal{R}}), \\ A_{in}^{ref} &= b_{out}(b_{out}^* - a_{out}^*) \equiv \mp i \sqrt{\mathcal{R}} (\sqrt{\mathcal{T}} \pm i \sqrt{\mathcal{R}}); \end{aligned} \quad (5)$$

or  $A_{in}^{ref} = \sqrt{\mathcal{R}} \exp(i\lambda)$  where  $\lambda = \mp \arctan \sqrt{\mathcal{T}(k)/\mathcal{R}(k)}$ ; for any value of  $k$  either the upper or lower sign is valid; hereinafter  $\mathcal{T}$ ,  $\mathcal{R}$  and  $\mathcal{J}$  denote the scattering parameters for the two-barrier system:  $\mathcal{T}(k) + \mathcal{R}(k) = 1$  for any value of  $k$ . It is evident that

$$A_{in}^{tr}(k) + A_{in}^{ref}(k) = 1, \quad |A_{in}^{tr}(k)|^2 + |A_{in}^{ref}(k)|^2 = 1. \quad (6)$$

Then, for  $a_1 \leq x \leq b_1$

$$\begin{aligned} \psi_{tr} &= a_{tr}^{(1)} \sinh[\kappa(x - b_1)] + b_{tr}^{(1)} \cosh[\kappa(x - b_1)], \\ \psi_{ref} &= a_{ref}^{(1)} \sinh[\kappa(x - b_1)] + b_{ref}^{(1)} \cosh[\kappa(x - b_1)]; \end{aligned} \quad (7)$$

for  $b_1 \leq x \leq x_c$

$$\begin{aligned} \psi_{tr} &= a_{tr}^{gap} \sin[k(x - x_c)] + b_{tr}^{gap} \cos[k(x - x_c)], \\ \psi_{ref} &= a_{ref}^{gap} \sin[k(x - x_c)]; \\ a_{tr}^{(1)} &= A_{in}^{tr} \frac{k}{\kappa} [P \cos(\phi) + Q \sin(\phi)] e^{ika_1}, \\ b_{tr}^{(1)} &= A_{in}^{tr} [Q \cos(\phi) - P \sin(\phi)] e^{ika_1}, \\ a_{ref}^{(1)} &= a_{ref}^{gap} \frac{k}{\kappa} \cos(\phi), \quad b_{ref}^{(1)} = -a_{ref}^{gap} \sin(\phi), \\ a_{tr}^{gap} &= A_{in}^{tr} P e^{ika_1}, \quad b_{tr}^{gap} = A_{in}^{tr} Q e^{ika_1} = b_{full}^{gap}, \\ a_{ref}^{gap} &= -2a_{out}^* b_{out} P e^{ika_1}. \end{aligned} \quad (8)$$

Of importance is to stress that

$$|\psi_{tr}(x_c - x, k)| = |\psi_{tr}(x - x_c, k)| \quad (9)$$

and, as a consequence,  $|d\psi_{tr}(x_c - x, k)/dx| = |d\psi_{tr}(x - x_c, k)/dx|$ . This is valid for any system of potential barriers, which possesses the mirror symmetry. It should be stressed however that  $\psi_{tr}^*(x_c - x, k) \neq \psi_{tr}(x - x_c, k)$  and  $\psi_{tr}(x_c - x, k) \neq \psi_{tr}(x - x_c, k)$ . The first inequality means that the 'weak-value' rule of fitting the initial state to the transmitted one "... by a parity flip combined with a time reversal..." [15] gives an incoming wave to be not linked causally to the outgoing transmitted one.

#### 4. Transmission and reflection as alternative subprocesses

To show that  $\psi_{tr}(x, k)$  and  $\psi_{ref}(x, k)$  define *alternative* subprocesses, let us proceed to the time-dependent scattering process described by the wave packet  $\Psi_{full}(x, t)$

$$\Psi_{full}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) \Psi_{full}(x, k) e^{-iE(k)t/\hbar} dk$$

which, at  $t = 0$ , represents, e.g., the Gaussian wave packet;

$$A(k) = (2l_0^2/\pi)^{1/4} \exp(-l_0^2(k - \bar{k})^2), \quad (10)$$

$$\int_{-\infty}^{\infty} |A(k)|^2 dk = 1, \quad \bar{x}_{full}(0) = 0, \quad \overline{x^2}_{full}(0) = l_0^2;$$

hereinafter, for any observable  $F$  and time-dependent state  $\psi$ ,  $\bar{F}_\psi(t)$  denotes the expectation value  $\frac{\langle \psi | \hat{F} | \psi \rangle}{\langle \psi | \psi \rangle}$ ; when  $\bar{F}_\psi(t)$  is constant its argument is omitted. Then the expression

$$\psi_{tr,ref}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) \psi_{tr,ref}(x, k) e^{-iE(k)t/\hbar} dk$$

gives the wave functions  $\psi_{tr}(x, t)$  and  $\psi_{ref}(x, t)$  to describe the time-dependent subprocesses. For any instant of time  $t$  (see (4))

$$\Psi_{full}(x, t) = \psi_{tr}(x, t) + \psi_{ref}(x, t). \quad (11)$$

A 1D completed scattering implies that the incident and scattered wave packets move in the asymptotically remote spatial regions where they do not interact with the system. At the first stage this process is described by the incident wave packet

$$\Psi_{full}(x, t) \simeq \Psi_{inc}^{full}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) \exp[i(kx - E(k)t/\hbar)] dk,$$

as the reflected wave packet has not yet appeared at this stage;  $a_1 \gg l_0$ . Accordingly, the transmission and reflection subprocesses are described at this stage by the wave packets

$$\begin{aligned} \psi_{tr} &\simeq \psi_{inc}^{tr} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) A_{in}^{tr}(k) \exp[i(kx - E(k)t/\hbar)] dk \\ \psi_{ref} &\simeq \psi_{inc}^{ref} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) A_{in}^{ref}(k) \exp[i(kx - E(k)t/\hbar)] dk \end{aligned}$$

In the  $k$ -representation it is easy to show that (see (5) and (13))

$$\bar{x}_{tr}^{inc}(0) = -\overline{\lambda'}^{inc}_{tr}, \quad \bar{x}_{ref}^{inc}(0) = -\overline{\lambda'}^{inc}_{ref}. \quad (12)$$

That is, the CMs of the wave packets  $\Psi_{full}(x, t)$ ,  $\psi_{tr}(x, t)$  and  $\psi_{ref}(x, t)$  start at  $t = 0$  from the different spatial points!

Similarly, for the final stage of scattering we obtain

$$\begin{aligned} \psi_{tr} &\simeq \psi_{out}^{tr} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) a_{out}(k) e^{i[k(x-D) - E(k)t/\hbar]} dk \\ \psi_{ref} &\simeq \psi_{out}^{ref} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) b_{out}(k) e^{i[k(2a_1-x) - E(k)t/\hbar]} dk. \end{aligned}$$

From the above it follows that

$$\begin{aligned} &\langle \psi_{inc}^{tr} | \psi_{inc}^{tr} \rangle + \langle \psi_{inc}^{ref} | \psi_{inc}^{ref} \rangle \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dx dk dk' A^*(k') A(k) e^{i[(k-k')x - (E(k) - E(k'))t/\hbar]} \\ &\quad \times [(A_{in}^{tr}(k'))^* A_{in}^{tr}(k) + (A_{in}^{ref}(k'))^* A_{in}^{ref}(k)]. \end{aligned}$$

Its integration by  $x$  and then by  $k'$ , with taking into account the second equality in (6), yields

$$\begin{aligned} & \langle \psi_{inc}^{tr} | \psi_{inc}^{tr} \rangle + \langle \psi_{inc}^{ref} | \psi_{inc}^{ref} \rangle \\ &= \int_{-\infty}^{\infty} |A(k)|^2 (|A_{in}^{tr}(k)|^2 + |A_{in}^{ref}(k)|^2) dk \\ &= \int_{-\infty}^{\infty} |A(k)|^2 dk \equiv \langle \Psi_{full} | \Psi_{full} \rangle = 1. \end{aligned}$$

That is, at the first stage, transmission and reflection obey the probabilistic "either-or" rule, what means that at this stage they are alternative subprocesses, despite interference between  $\psi_{inc}^{tr}(x, t)$  and  $\psi_{inc}^{ref}(x, t)$ . This interference is such that

$$\begin{aligned} \langle \psi_{inc}^{tr} | \psi_{inc}^{ref} \rangle &= \int_{-\infty}^{\infty} |A(k)|^2 (A_{in}^{tr}(k))^* A_{in}^{ref}(k) dk \\ &= i \int_{-\infty}^{\infty} |A(k)|^2 f(k) \sqrt{\mathcal{T}(k)\mathcal{R}(k)} dk; \end{aligned}$$

here  $f(k)$  is the piece-wise constant function to take two values, 1 or  $-1$  (see (5)); its change on the  $k$ -scale occurs at the points where  $\mathcal{R}(k) = 0$ . So that  $\langle \psi_{inc}^{tr} | \psi_{inc}^{ref} \rangle + \langle \psi_{inc}^{ref} | \psi_{inc}^{tr} \rangle = 0$ !

We have also to stress that (see (5))

$$|A_{in}^{tr}(k)|^2 = |a_{out}(k)|^2 = \mathcal{T}(k), \quad |A_{in}^{ref}(k)|^2 = |b_{out}(k)|^2 = \mathcal{R}(k),$$

and hence

$$\langle \psi_{inc}^{tr} | \psi_{inc}^{tr} \rangle = \langle \psi_{out}^{tr} | \psi_{out}^{tr} \rangle \equiv \mathbf{T}, \quad \langle \psi_{inc}^{ref} | \psi_{inc}^{ref} \rangle = \langle \psi_{out}^{ref} | \psi_{out}^{ref} \rangle \equiv \mathbf{R}$$

All this means that a 1D completed scattering is a complex process to consist of two alternative subprocesses not only at its *final* stage, when the transmitted and reflected wave packets move in the nonoverlapping spatial regions, but also at its *first* one to precede the stage of interaction of the wave packet with the barriers. The number of particles taking part in either subprocess, at the first and final stages of scattering, is the same. And this is valid for any symmetric system of potential barriers and any shape of the incident wave packet.

At the stage of interaction we meet a more complex situation. The norm  $\mathbf{R}$  is constant at this stage too! This follows eventually from the fact that  $\psi_{ref}(x, k)$ , unlike  $\psi_{tr}(x, k)$ , is *zero* at the point  $x_c$ . As to  $\mathbf{T}$ , this is valid only in the limit of narrow in  $k$ -space wave packets. In the general case,  $\mathbf{T}$  varies at this stage (the continuity equation is nonlinear and, thus, the continuity of the probability current density for a single wave does not guarantee that this is so for the wave packet). However, in our numerical calculations for  $l_0 \sim D$ , the deviation of  $\mathbf{T}$  from the constant value  $1 - \mathbf{R}$  has amounted several percentages only.

Now, when the time evolution of either subprocess is known, we can proceed to studying their temporal aspects. In doing so we have to take into account that in the general case the present quantum model allows only the *asymptotic* transmission time to be defined strictly. The *local* one can be strictly defined only for sufficiently narrow in  $k$ -space wave packets. In the general case the model allows an approximate estimation of this quantity, because the maximal deviation of  $\mathbf{T}$  from the constant value  $1 - \mathbf{R}$  is not large at the stage of interaction. The estimation accuracy can be controlled by this deviation.

## 5. The phase and dwell times of the CQM

For the following it is useful to present also the phase and dwell times to describe the two-barrier system within the CQM. We begin with the Wigner phase time  $\tau_{ph}$  (see [1, 2]) written in the notations of the transfer matrix approach [17]. In this case  $a_{out} \equiv \sqrt{\mathcal{T}} \exp(i\mathcal{J})$ , and hence

$$\tau_{ph}(k) = \frac{m}{\hbar k} \cdot \mathcal{J}'(k);$$

$$\mathcal{J}' = J' + \frac{\mathcal{T}}{T^2} [(1 - R^2)(J' + l) + T' \sin(2(J + kl))]$$

$$J' = \frac{T}{\kappa} [\theta_{(+)}^2 \sinh(2\kappa d) + \theta_{(-)} \kappa d],$$

$$T' = 2\theta_{(+)}^2 \frac{T^2}{\kappa} [2\theta_{(-)} \sinh^2(\kappa d) + \kappa d \sinh(2\kappa d)];$$

hereinafter, the prime denotes the derivative on  $k$ .

The Büttiker dwell time  $\tau_D$  reads as

$$\tau_D = \frac{m}{\hbar k} \int_{a_1}^{b_2} |\Psi_{full}(x, k)|^2 dx.$$

For the system,  $\tau_D = \tau_{full}^{(1)} + \tau_{full}^{gap} + \tau_{full}^{(2)}$ ; here  $\tau_{full}^{(1)}$ ,  $\tau_{full}^{gap}$  and  $\tau_{full}^{(2)}$  describe the intervals  $[a_1, b_1]$ ,  $[b_1, a_2]$  and  $[a_2, b_2]$ , respectively:

$$\begin{aligned} \tau_{full}^{(1)} &= \left\{ 2\kappa d [(\kappa^2 - k^2)(1 + R) + 2k_0^2 \sqrt{R} \sin(J + kl)] \right. \\ &\quad + \left[ k_0^2(1 + R) + 2(\kappa^2 - k^2) \sqrt{R} \sin(J + kl) \right] \sinh(2\kappa d) \\ &\quad \left. + 8\kappa k \sqrt{R} \cos(J + kl) \sinh^2(\kappa d) \right\} \frac{m\mathcal{T}}{4\hbar k \kappa^3 T}, \\ \tau_{full}^{gap} &= \frac{m\mathcal{T}}{\hbar k^2 T} [kl(1 + R) + 2\sqrt{R} \sin(J + kl) \sin(kl)], \\ \tau_{full}^{(2)} &= \frac{m\mathcal{T}}{4\hbar k \kappa^3} [2\kappa d(\kappa^2 - k^2) + k_0^2 \sinh(2\kappa d)]; \\ k_0 &= \sqrt{2mV_0/\hbar}. \end{aligned}$$

As was said above,  $\tau_{ph}$  and  $\tau_D$  give neither local nor asymptotic transmission times.

## 6. The local and asymptotic group scattering times

Our next step is to present local and asymptotic group scattering times for the subprocesses. For example, the local group transmission time  $\tau_{tr}^{loc}$  (see [16]), which describes the motion of the CM of the wave packet  $\psi_{tr}(x, t)$  in the barrier region, reads as follows:  $\tau_{tr}^{loc} = t_{exit} - t_{entry}$ , where  $t_{entry}$  and  $t_{exit}$  are such instants of time that

$$\bar{x}_{tr}(t_{entry}) = a_1; \quad \bar{x}_{tr}(t_{exit}) = b_2.$$

That is, the concept of the local *group* transmission time represents the average time spent by transmitted particles in the interval  $[a_1, b_2]$  as the time spent in this region by the CM of the wave packet  $\psi_{tr}(x, t)$ .

Of course, this quantity does not give a complete description of the temporal aspects of transmission, as the system affects the subensemble of transmitting particles not only when the CM of  $\psi_{tr}(x, t)$  moves within the region  $[a_1, b_2]$ . One has also to

introduce the *asymptotic* group transmission time to describe the CM's dynamics in the large spatial region  $[a_1 - L, b_2 + L]$  where  $L \gg l_0$ . When the CM of the wave packet  $\psi_{tr}(x, t)$  is at the extreme points  $a_1 - L$  and  $b_2 + L$  this packet does not interact with the system (note, for a 1D *completed* scattering, the velocity of widening this wave packet is assumed to be small in comparison with the CM's velocity).

All this means that the asymptotic transmission time can be introduced via the transmitted  $\psi_{tr}^{tr}$  and to-be-transmitted  $\psi_{tr}^{tr}$  wave packets. Making use of the  $k$ -representation, for the first stage of scattering we obtain

$$\bar{x}_{tr}(t) \simeq \bar{x}_{tr}^{inc}(t) = \frac{\hbar \bar{k}_{tr}}{m} t - \overline{\lambda'(k)_{tr}}^{inc}; \quad (13)$$

similarly, at the final stage

$$\bar{x}_{tr}(t) \simeq \bar{x}_{tr}^{out}(t) = \frac{\hbar \bar{k}_{tr}}{m} t - \overline{\mathcal{J}'(k)_{tr}}^{out} + D.$$

We have to stress that  $\bar{k}_{tr}^{out} = \bar{k}_{tr}^{inc} = \bar{k}_{tr}$ .

From the above it follows that the time  $\tau_{gr}^{tr}(L)$  spent by the CM in the spatial region  $[a_1 - L, b_2 + L]$  is

$$\tau_{gr}^{tr}(L) \equiv t_r - t_l = \frac{m}{\hbar \bar{k}_{tr}} \left( \overline{\mathcal{J}'(k)_{tr}}^{out} - \overline{\lambda'(k)_{tr}}^{inc} + 2L \right),$$

where  $t_r$  and  $t_l$  obey the equations

$$\bar{x}_{tr}^{inc}(t_l) = a_1 - L; \quad \bar{x}_{tr}^{out}(t_r) = b_2 + L.$$

The quantity  $\tau_{as}^{tr} = \tau_{gr}^{tr}(0)$  is just the asymptotic (extrapolated) group transmission time,

$$\tau_{as}^{tr} = \frac{m D_{eff}^{tr}}{\hbar \bar{k}_{tr}}, \quad D_{eff}^{tr} = \overline{\mathcal{J}'(k)_{tr}}^{out} - \overline{\lambda'(k)_{tr}}^{inc}. \quad (14)$$

Similarly, for reflection on the *symmetric* system we obtain

$$\tau_{as}^{ref} = \frac{m D_{eff}^{ref}}{\hbar \bar{k}_{ref}}, \quad D_{eff}^{ref} = \overline{\mathcal{J}'(k)_{ref}}^{out} - \overline{\lambda'(k)_{ref}}^{inc}. \quad (15)$$

Quantities  $D_{eff}^{tr}$  and  $D_{eff}^{ref}$  in (14) and (15) can be treated as effective widths of the system for transmitted and reflected particles, respectively. Note also that  $\bar{k}_{ref}^{inc} = -\bar{k}_{ref}^{out} = \bar{k}_{ref}$ .

For narrow in the  $k$ -space wave packets

$$\tau_{as}^{tr}(k) = \tau_{as}^{ref}(k) = \tau_{as}(k) = \frac{m D_{eff}(k)}{\hbar k}, \quad D_{eff}(k) = \mathcal{J}'(k) - \lambda'(k).$$

Correspondingly, for the starting points  $\bar{x}_{tr}^{inc}(0)$  and  $\bar{x}_{ref}^{inc}(0)$  (see (12)) we have  $\bar{x}_{tr}^{inc}(0) = \bar{x}_{ref}^{inc}(0) = x_{start} = -\lambda'(k)$ . In the general case explicit expressions for these quantities are cumbersome. However, for  $l = 0$  when the two-barrier system transforms into the  $D$ -wide rectangular barrier, we have for  $E \leq V_0$  (see [16])

$$D_{eff}(k) = \frac{4}{\kappa} \frac{[k^2 + \kappa_0^2 \sinh^2(\kappa D/2)] [\kappa_0^2 \sinh(\kappa D) - k^2 \kappa D]}{4k^2 \kappa^2 + \kappa_0^4 \sinh^2(\kappa D)}; \quad (16)$$

$$x_{start}(k) = -2 \frac{\kappa_0^2}{\kappa} \frac{(\kappa^2 - k^2) \sinh(\kappa D) + k^2 \kappa D \cosh(\kappa D)}{4k^2 \kappa^2 + \kappa_0^4 \sinh^2(\kappa D)}$$

So, the key feature to differ  $\tau_{ph}(k)$  from  $\tau_{as}(k)$  is as follows. The former is based on the unwarranted assumption that  $x_{start} = \bar{x}_{full}^{inc}(0) = 0$ , resulting in  $D_{eff}(k) = \mathcal{J}'(k)$ . The latter implies that  $x_{start} = -\lambda'(k) \neq \bar{x}_{full}^{inc}(0)$  and  $D_{eff}(k) = \mathcal{J}'(k) - \lambda'(k)$ .

This difference becomes striking for the  $\delta$ -potentials. In this case  $\mathcal{J}'(k) \equiv \lambda'(k)$  (see [16]). As a result, by the CQM

$$x_{start} = 0, \quad D_{eff}(k) = \mathcal{J}'(k);$$

however, by the present model

$$x_{start} = -\mathcal{J}'(k), \quad D_{eff}(k) = 0.$$

## 7. The dwell times for transmission and reflection

The dwell times for transmission and reflection –  $\tau_D^{tr}$  and  $\tau_D^{ref}$  – are defined for the two-barrier system as follows

$$\tau_D^{tr} = \frac{m}{\hbar k \mathcal{T}} \int_{a_1}^{b_2} |\psi_{tr}(x, k)|^2 dx \equiv \tau_{tr}^{(1)} + \tau_{tr}^{gap} + \tau_{tr}^{(2)},$$

$$\tau_D^{ref} = \frac{m}{\hbar k \mathcal{R}} \int_{a_1}^{x_c} |\psi_{ref}(x, k)|^2 dx \equiv \tau_{ref}^{(1)} + \tau_{ref}^{gap};$$

here  $\tau_{tr}^{(1)}$  and  $\tau_{ref}^{(1)}$  describe the interval  $[a_1, b_1]$ ;  $\tau_{tr}^{gap}$  and  $\tau_{ref}^{gap}$  characterize the gap  $[b_1, a_2]$ ;  $\tau_{tr}^{(2)}$  relates to  $[a_2, b_2]$ .

Calculations yield

$$\tau_{tr}^{(1)} = \tau_{tr}^{(2)} = \frac{m}{2\hbar k \kappa^3} \left[ 2\kappa d(\kappa^2 - k^2) + k_0^2 \sinh(2\kappa d) \right],$$

$$\tau_{tr}^{gap} = \frac{m}{\hbar k^2 T} \left[ k l (1 + R) + 4 \sqrt{R} \sin\left(\frac{kl}{2}\right) \sin\left(J + \frac{kl}{2}\right) \right],$$

$$\tau_{ref}^{(1)} = \left\{ 2\kappa d \left[ \kappa^2 - k^2 - k_0^2 \cos(kl) \right] + 4k\kappa \sin(kl) \sinh^2(\kappa d) \right. \\ \left. + \left[ k_0^2 - (\kappa^2 - k^2) \cos(kl) \right] \sinh(2\kappa d) \right\} \frac{m \mathcal{T}}{2\hbar k \kappa^3} |P|^2,$$

$$\tau_{ref}^{gap}(k) = \frac{m \mathcal{T}}{\hbar k^2} |P|^2 \left[ kl - \sin(kl) \right];$$

$$\mathcal{T}^{-1} = 1 + 4 \frac{R}{T^2} \cos^2(J + kl),$$

$$|P|^2 = \frac{1}{T} \left[ 1 + R - 2 \sqrt{R} \sin(J + kl) \right].$$

Note,  $\tau_{tr}^{(2)} = \tau_{tr}^{(1)} \equiv \tau_{tr}^{bar}$ . Besides, if  $\tau_{tr}^{left}$  and  $\tau_{tr}^{right}$  denote, respectively, the transmission dwell times for the intervals  $[a_1, x_c]$  and  $[x_c, b_2]$ , then the following equality is valid (see (9)),

$$\tau_{tr}^{left} = \tau_{tr}^{right} = \tau_{tr}^{bar} + \tau_{tr}^{gap}/2 = \tau_D^{tr}/2. \quad (17)$$

That is, this model unlike the CQM obeys the natural requirement: for any barrier structure to possess the mirror symmetry, the transmission time must be the same for its two reflection symmetric parts. Note,  $\tau_{as}^{tr}(k) = \tau_{as}^{ref}(k)$ , but  $\tau_D^{tr}(k) \neq \tau_D^{ref}(k)$ !

## 8. On the Hartman effect

As is known (see [1, 2] and references therein, as well as [8]) the generalized Hartman effect appears in the CQM in the

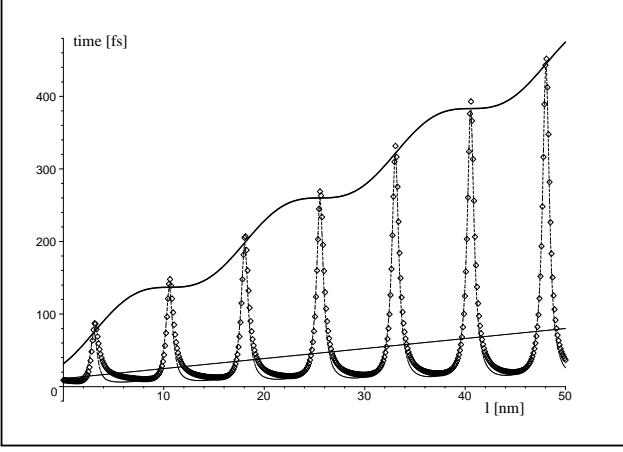


Figure 1: The dependence of  $\tau_D^{tr}$  (full curve),  $\tau_D$  (broken curve),  $\tau_{ph}$  (open circles) and  $\tau_{free}$  (straight line) on the distance  $l$  between the barriers

opaque limit:  $\kappa d \rightarrow \infty$  and  $\cos^2(J + kl) \gg T^2/(4R)$ . We consider two cases. Let firstly  $d \rightarrow \infty$  but  $V_0$  be fixed. Omitting the exponentially small terms, we obtain

$$\begin{aligned} \tau_{tr}^{gap} &\approx \frac{m\theta_{(+)}^2}{2\hbar k^2} \left[ kl + 2 \sin\left(\frac{kl}{2}\right) \sin\left(J_{(\infty)} + \frac{kl}{2}\right) \right] e^{2\kappa d}, \\ \tau_{tr}^{(1)} = \tau_{tr}^{(2)} &\approx \frac{mk_0^2}{4\hbar k \kappa^3} e^{2\kappa d}, \quad J_{(\infty)} = \arctan(\theta_{(-)}); \\ \tau_D^{ref} &\approx \tau_{ref}^{(1)} \approx \frac{m}{2\hbar k \kappa^3 \theta_{(+)}^2} \cdot \frac{1}{1 + \sin(J_{(\infty)} + kl)} \\ &\quad \times [k_0^2 - (\kappa^2 - k^2) \cos(kl) + k\kappa \sin(kl)]; \\ \tau_{ph} &\approx \frac{2m}{\hbar k \kappa}; \quad \tau_D \approx \tau_{full}^{(1)} \approx \frac{m}{\hbar k \kappa^3 \theta_{(+)}^2 \cos^2(J_{(\infty)} + kl)} \\ &\quad \times [k_0^2 + (\kappa^2 - k^2) \sin(J_{(\infty)} + 2kl) + 2k\kappa \cos(J_{(\infty)} + 2kl)] \quad (18) \end{aligned}$$

Another variant of the opaque limit is realized when  $V_0 \rightarrow \infty$  (or  $k_0 \rightarrow \infty$ ) but  $d$  is fixed. Again, omitting the exponentially small terms, we obtain

$$\begin{aligned} \tau_{tr}^{gap} &\approx \frac{mk_0^2}{8\hbar k^4} [kl - \sin(kl)] e^{2k_0 d}, \\ \tau_{tr}^{(1)} = \tau_{tr}^{(2)} &\approx \frac{m}{4\hbar k k_0} e^{2k_0 d}, \quad \tau_D^{ref} \approx \tau_{ref}^{(1)} \approx \tau_D, \\ \tau_{ph} &\approx \frac{2m}{\hbar k k_0}, \quad \tau_D \approx \tau_{full}^{(1)} \approx \frac{2mk}{\hbar k_0^3}. \quad (19) \end{aligned}$$

Note that in this case

$$\frac{\tau_{tr}^{gap}}{\tau_{tr}^{(1)}} \approx \frac{k_0^3}{2k^3} [kl - \sin(kl)]. \quad (20)$$

As is seen from Exps. (18) and (19), far from resonant points where  $\mathcal{T}(k) = 1$  both the phase time  $\tau_{ph}$  and the dwell time  $\tau_D$  saturate in the limit  $d \rightarrow \infty$  ( $V_0$  is fixed) and diminish in the limit  $V_0 \rightarrow \infty$  ( $d$  is fixed). However, such behavior of these quantities has no physical content, because both are neither local nor asymptotic transmission times.

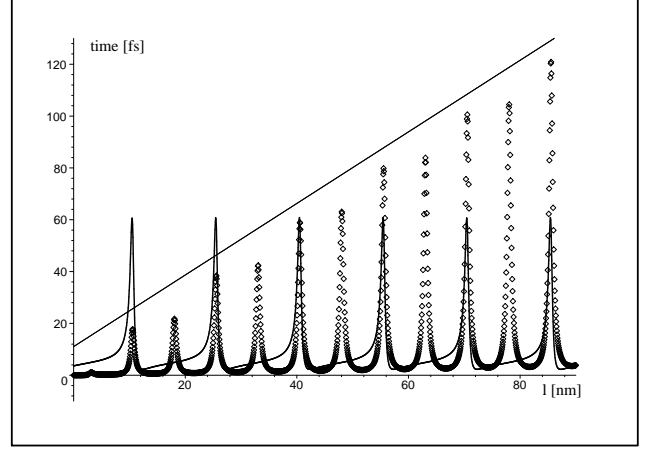


Figure 2: The dependence of  $\tau_{ref}^{(1)}$  (full curve),  $\tau_{ref}^{gap}$  (open circles) and  $\tau_{free}$  (straight line) on the distance  $l$  between the barriers

As regards  $\tau_D^{tr}$  to play the role of the local transmission time, this quantity increases exponentially in the opaque limit (see Exps. (18) and (19)). In this case a tunneling particle spends, on the average, the same time in the regions  $[a_1, b_1]$  and  $[a_2, b_2]$  occupied by the identical barriers. Besides, as is seen from Exp. (20), the probability to find a tunneling particle in the space between the barriers increases in the limit  $V_0 \rightarrow \infty$ , as compared with that of finding its in the barrier regions.

Fig. 1 shows the dependence of  $\tau_D^{tr}$ ,  $\tau_D$ ,  $\tau_{ph}$  as well as  $\tau_{free}$ ,  $\tau_{free} = m(2d + l)/(\hbar k)$ , on the distance  $l$  between the barriers;  $m = 0.067m_e$ ,  $E = 0.1\text{eV}$ ,  $V_0 = 0.15\text{eV}$ ,  $d = 4\text{nm}$ ;  $m_e$  is the electron mass. As was expected, the times  $\tau_D^{tr}$  and  $\tau_D$  are equal at the resonant points (both differ from the phase time  $\tau_{ph}$  in this case). Of importance is that  $\tau_D^{tr}$ , unlike  $\tau_D$  and  $\tau_{ph}$ , is everywhere a nondecreasing function of  $d$  and  $l$ .

As regards the reflection dwell time  $\tau_D^{ref}$ , it behaves like  $\tau_D$  in the opaque limit. However, such result does not lead to any conflict with relativity theory. It says simply that, in this limit, the average depth of penetration of reflected particles into the spatial region occupied by the system either diminishes ( $V_0 \rightarrow \infty$ ,  $d$  is fixed) or saturates ( $d \rightarrow \infty$ ,  $V_0$  is fixed). Another situation arises at resonant points. As is seen from Fig. 2 (all parameters for Figs. 1 and 2 are the same), under resonance conditions the time  $\tau_{ref}^{gap}$  increases linearly when  $l$  grows, but  $\tau_{ref}^{(1)}$  remains constant in this case because  $d$  is fixed. What is interesting is that the time  $\tau_{ref}^{(1)}$  is large enough at the resonant points where  $\sin(J + kl) = -1$ ; for  $\sin(J + kl) = 1$  it is small.

Note that like  $\tau_{ph}$ , the asymptotic transmission time  $\tau_{as}^{tr}$  saturates in the limit  $d \rightarrow \infty$ . However, this fact does not at all mean that the effective velocity of a tunneling particle becomes superluminal in this case. It is demonstrated by Fig. 3 which shows the function  $\bar{x}_{tr}(t)$  to describe scattering the Gaussian wave packet (10) on the rectangular potential barrier (i.e.,  $l = 0$ ):  $l_0 = 10\text{nm}$ ,  $\bar{E} = (\hbar k)^2/2m = 0.05\text{eV}$ ,  $a_1 = 200\text{nm}$ ,  $b_2 = 215\text{nm}$ ,  $V_0 = 0.2\text{eV}$ . (Note, in this case the deviation of  $\mathbf{T}$  from  $1 - \mathbf{R}$  does not exceed five percentages, though the wave-packet's and barrier's widths are of the same order.)

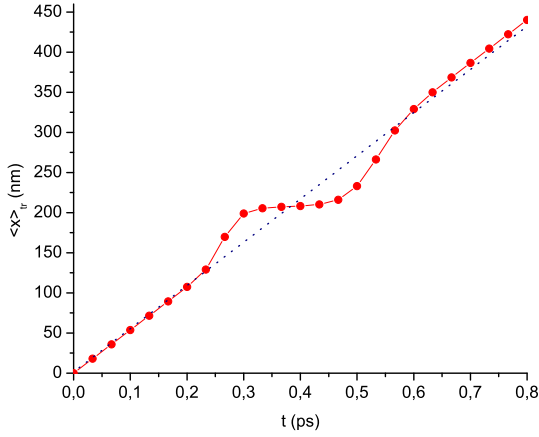


Figure 3: The CM's positions for  $\psi_{tr}(x, t)$  (circles) and for the corresponding RWP (dashed line) as functions of time  $t$ .

This figure shows explicitly a qualitative difference between the local and asymptotic group times. While the former gives the time spent by the CM of this packet just in the barrier region, the latter describes the influence of the barrier on the CM in the course of the whole process. More precisely,  $\tau_{as}^r - \tau_{free}^r$  is the time delay acquired by transmitted particles in the course of a 1D *completed* scattering;  $\tau_{free}^r = mD/\hbar k_0$ . It describes the relative motion of the CMs of the *transmitted* wave packet and the corresponding freely moving RWP. The latter starts from the point  $\bar{x}_{tr}^{inc}(0)$ , rather than from  $\bar{x}_{full}^{inc}(0)$  (as is seen from (16), in the opaque limit,  $\bar{x}_{tr}^{inc}(0) \approx \bar{x}_{full}^{inc}(0) = 0$ ). In the case considered,  $\tau_{loc}^r \approx 0,155 ps$ ,  $\tau_{as}^r \approx 0,01 ps$ ,  $\tau_{free}^r \approx 0,025 ps$ .

Thus, the influence of the opaque rectangular barrier on the transmitted wave packet has a complicated character. Both the local group transmission time and the dwell one say that the barrier retards the motion of the CM when it enters the barrier region. The asymptotic group transmission time tells us that the total influence of the opaque barrier on the transmitted wave packet has an accelerating character: at the final stage of a 1D completed scattering this packet moves ahead the RWP.

Note that this effect relates to the asymptotically large spatial interval, and it does not contradict special relativity. Let  $v_{tr}(\bar{E})$  ( $v_{tr}(\bar{E}) \ll c$ ) be the CM's velocity at the first and final stages of scattering. Then the effective velocity  $v_{eff}(\bar{E})$  of the CM in the region  $[a_1 - L, b_2 + L]$  can be estimated from the equality

$$\tau_{gr}^r(L) = \frac{2L + D_{eff}(\bar{E})}{v_{tr}(\bar{E})} \equiv \frac{2L + D}{v_{eff}(\bar{E})}.$$

In the opaque limit  $D_{eff} \rightarrow 2/\kappa(\bar{E})$  (see (16)). Hence with the inequalities  $L \gg l_0 \gg D$  to secure the opaque limit in the time-dependent case,  $v_{eff}(\bar{E}) \approx v_{tr}(\bar{E})$ , when  $\kappa(\bar{E})L \gg 1$ .

So, the saturation of the *asymptotic* transmission time and the exponential increase of the *local* one can be simultaneously observed if only  $\bar{E} < V_0$  and  $L \gg l_0 \gg D$ . To keep the last condition  $l_0 \gg D$  is very important. Otherwise the transmitted

wave packet is built of the 'above-barrier' harmonics what leads to the violation of the regime of an opaque barrier. When  $l \neq 0$ , the regime of an opaque two-barrier system is violated if the parameters  $d$ ,  $V_0$  and  $l_0$  are fixed but  $l \rightarrow \infty$ . In this case the function  $\mathcal{T}(k)$  is rapidly varying one. Thus, in order to secure this regime in this limit,  $l_0$  must increase proportionally to  $l$ .

## 9. Final remarks

So, to reconcile the quantum description of a 1D completed scattering with probability theory and special relativity, we have developed its new quantum model to treat this process as a complex one to consist of two alternative subprocesses – transmission and reflection. The model obeys the next two requirements: (a) to be consistent with *probability theory*, it avoids any averaging over statistical data associated with alternative subprocesses, giving their individual description at all stages of scattering; (b) to be consistent with *special relativity*, it ensures the causal character of their evolution at all stages of scattering.

As is shown, the stationary wave function to describe this process uniquely splits into two components, each having one incoming and one outgoing waves joined at some spatial point with keeping the continuity of each (complex-valued) subprocess's wave function as well as the continuity of the corresponding probability current density. For any symmetric potential barrier the joining point coincides with the midpoint of the barrier region, irrespective of the particle's energy  $E$ . By this model, reflected particles never cross this point.

A complete description of the temporal aspects of either subprocess needs the introduction of local and asymptotic scattering times. The first ones give the average time spent by transmitted (or reflected) particles in the barrier region. The second ones relate directly to the time delay caused by the influence of the potential barrier on transmitted (or reflected) particles in the course of the whole time-dependent scattering process.

The CQM to violate the above consistency requirements is unable to give such a detailed description. As a result, none of the scattering time concepts introduced in this model, including the phase time  $\tau_{ph}$  and the dwell time  $\tau_D$ , can be considered as well-established. For example, the fact that "...the measured delay times agree with the theoretical predictions provided by the method of stationary phase..." (see [14] as well as [1, 2, 4]) does not at all establish  $\tau_{ph}$ . This agreement means nothing but both are based on the same time-keeping procedure. This agreement is misleading, because this time-keeping procedure is internally inconsistent, having no relation both to the local transmission time and to the asymptotic one.

In this paper we present the explicit expressions for the local and asymptotic *group* scattering times as well as for the (local by definition) *dwell* transmission and reflection times to describe a particle scattering on the system of two identical rectangular barriers. Both the local transmission times increase exponentially in the limit  $d \rightarrow \infty$ , but the asymptotic group transmission time saturates in this case. So that the Hartman effect relates only to the *asymptotic* group transmission time.

At first glance, these quantities cannot be measured, because the time evolution of either subprocess is latent in the region

$x < x_c$ . However, the situation when some physical quantity admits direct measurements is rare exception, rather than rule.

We have to draw reader's attention on the fact that the presented decomposition of the probability wave  $\Psi_{inc}^{full}(x, k)$  into  $\psi_{inc}^{tr}(x, k)$  and  $\psi_{inc}^{ref}(x, k)$  is analogous to the decomposition of the incident plane-polarized monochromatic electromagnetic wave, in the Faraday effect of classical electrodynamics, into two circular-polarized incident waves: (a) in either case the decomposition is unique – the pair of the circular-polarized electromagnetic waves is unique for a given magnetic field; the pair of the probability waves  $\psi_{inc}^{tr}$  and  $\psi_{inc}^{ref}$  is unique for a given potential barrier; (b) in either case the paired waves are latent at the first stage of scattering; (c) in either case the unambiguous interpretation of experimental data for scattered waves is impossible without addressing the corresponding theoretical model to give uniquely the time evolution of waves at all stages of scattering. Of course, the latter implies the consistency of either model.

All the above fully concerns the study of the temporal aspects of tunneling within the CQM, as none of the experimental time-keeping procedures developed within this model implies only direct measurements. Each of them is based also on assumptions borrowed from the theoretical concept tested. For example, the procedure of measuring the phase time is based, like this concept itself, on the assumption that the role of the RWP is played by the wave packet  $\Psi_{inc}^{full}(x, t)$ . However, as was shown above, this wave packet is not linked causally to the transmitted one  $\psi_{out}^{tr}(x, t)$ . Thus, strictly speaking, it is meaningless to speak of any experimental realization of the phase time, because the physical meaning of this quantity is ambiguous.

By the concept of the asymptotic group transmission time  $\tau_{as}$ , the role of the RWP, in measuring the duration of a 1D completed scattering for transmitted particles, must be played by the wave packet  $\psi_{inc}^{tr}(x, t)$  rather than  $\Psi_{inc}^{full}(x, t)$ . Unlike the phase time concept, that of  $\tau_{as}$  can be assumed as a basis of the corresponding experimental time-keeping procedure consistent with the causality principle.

As regards the dwell transmission time  $\tau_D^{tr}$ , this quantity is simply the doubled transmission dwell time  $\tau_{tr}^{right}$ . Thus, measuring  $\tau_D^{tr}$  is reduced in fact to the experimental verification (e.g., with the help of the Larmor clock procedure) of the equality  $\tau_{tr}^{left}(k) = \tau_{tr}^{right}(k)$  (see (17)) to reflect the symmetry of the system, as well as to measuring the quantity  $\tau_{tr}^{right}$  to describe the region  $[x_c, b_2]$  where there is no interference.

As is seen, our approach discards the existing scattering time concepts. Notwithstanding it well respects the ideas and clock models to underlie them. It says only that they must be applied to the subprocess's wave functions. In this paper we show how to do this by the example of the time-keeping procedures to underlie Wigner's and Büttiker's scattering time concepts. However, it is desirable also to realize this programm for other time-keeping procedures: on the one hand, this can made these procedures consistent with classical probability theory and special relativity; on the other hand, this can shed new light on the time evolution of the transmission and reflection subprocesses.

It is worth to dwell separately on the Feynman, Bohmian and Wigner approaches to introduce one-particle trajectories. As

it follows from our model,  $\psi_{ref}(x, t) \equiv 0$  for  $x > x_c$ . This means that the incident particle to have crossed the midpoint  $x_c$  of the symmetric barrier cannot return backward into the region  $x < x_c$ . This result agrees with classical mechanics. Indeed, for a classical particle to impinge from the left on a smooth symmetrical potential barrier, the midpoint of its barrier region is the extreme right turning point, irrespective of the particles mass and the barrier's form and size. That is, in fact our approach extends this property onto quantum particles. Thus, by classical mechanics and this quantum approach, all one-particle trajectories to violate the above restriction must be discarded as noncausal. To avoid the appearance of such Feynman (as well Wigner and Bohmian) trajectories for a 1D completed scattering, they must be adapted to (derived from) the wave functions to describe its subprocesses.

From the above reasoning it follows also that the unusual, piecewise-continues character of the wave functions  $\psi_{tr}(x, k)$  and  $\psi_{ref}(x, k)$  reflects simply the properties of the context (complex of real physical conditions) under which both the subensembles of particles move (see [6]). From the viewpoint of classical probability theory we deal here with a complex (discontinuous) context to differ cardinally in the regions  $x < x_c$  and  $x > x_c$ . This context creates each subensemble, in these regions, in different Schrödinger states joined at the point  $x_c$  with keeping the causal character of the time evolution of this subensemble.

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